ABSTRACT

This paper presents several post-FFT fine frame synchronization algorithms in OFDM systems. Two different approaches are followed: the first is a channel-blind approach, which does not take into consideration the effect of the channel. The second is a channel-parametric approach, which is a novel in that it employs simple models in order to approximate the effect of the unknown channel on a set of post-FFT frame samples. The proposed algorithms are evaluated and compared through simulations for different types of channels. It is shown that they can produce reliable estimates even for one dedicated OFDM symbol only. On the other hand, it is shown that the reliability of the estimates depends on the channel characteristics.

I. INTRODUCTION

OFDM is an air-interface scheme under intense current development and evaluation, particularly in view of it’s adoption in DVB-T [1], HYPERLAN-II [2] and IEEE P802.11a [3]. It may also be the natural choice for 4G cellular and beyond. Synchronization in OFDM systems is one of the most difficult and demanding tasks. For time and frequency synchronization purposes, correlation-based methods are utilized over preambles that consist of periodic signal repetitions. These methods are appropriate for coarse frame synchronization only, as they are easily affected by the channel spread and AWGN [4]-[6]. Thus, a second and more refined frame (i.e., fine) synchronization stage is needed. The frame synchronization algorithms identify the arrival time of the first symbol of the frame. Thus, these methods are found in the literature as fine symbol synchronization methods also.

Several approaches for fine frame synchronization have been proposed, most of them in the frequency domain [7]-[9]. These algorithms manipulate the linear (over the frequency index) phase rotation induced by the time delay on the post-FFT observables [7]-[8]. All these fine synchronization algorithms are channel-blind and produce reliable estimates for an ideal or “mild” channel, which does not itself introduce significant amount of rotation to the post-FFT observables. Channels that incur significant such rotations lead to biased estimates and thus systematic errors, which translate to a degradation of system performance. For this reason the channel-parametric approach is introduced. In this approach the unknown effect of the channel is approximated by the use of simple models. Other non-linear channel-blind methods are also explored and evaluated. These methods use the post-FFT observables in order to identify the taps of the Channel Impulse Response (CIR) and then identify the arrival of the first significant energy tap.

This paper is organized as follows: it describes the observation model in Section II. The algorithmic description of the post-FFT synchronization algorithms is given in Section III. A channel blind algorithm, which efficiently performs a joint phase unwrapping and estimates the slope of the linear rotation, is proposed. Moreover, channel-parametric algorithms are proposed. Non-linear channel-blind methods are described in Section III. The robustness and the performance of the proposed algorithms are evaluated through simulations for three different types of channels in Section IV.

II. OBSERVATION MODEL

In an OFDM system the information is encoded into N-QAM or PSK symbols. After a serial-to-parallel conversion, they are fed into an N-length transmitter IFFT, whose output is parallel-to-serial converted and, after the addition of a Cyclic Prefix (CP) of length \( \nu \) (sufficiently long with respect to the channel spread), it is converted into an analog signal for transmission through the air. At the receiver the reverse process takes place. After the analog-to-digital conversion the cyclic prefix is removed, the signal is converted to parallel format and is passed through an N-point FFT.

1 This work has been partially supported by ADAMAS (IST-1999-10731) and WIND-FLEX (IST-1999-10025) projects, funded by the EU.
In the case of a time error $\varepsilon$ (and no other impairment) which is an integer multiple of the sampling time and which is such that the circularity of the OFDM symbol is maintained (i.e., the delayed symbol is a circularly shifted version of the transmitted) the observables at the output of an OFDM receiver can be computed as

$$Y_m(k) = X_m(k)H_m(k)e^{-j\alpha k} + n(k), \quad 0 \leq k \leq N - 1$$

where $a = \frac{2\pi}{N}$ where $N$ is the FFT size, $X_m(k)$ is the transmitted QAM symbol loaded on the $k$-th sub-carrier of the $m$-th transmitted OFDM symbol, $Y_m(k)$ is the corresponding observable, $H_m(k)$ is the $k$-th sample of the frequency response of the channel (which is assumed to be static over one OFDM symbol duration), and $n(k)$ is the noise term.

### III. ALGORITHMIC DESCRIPTION

A coarse frame synchronization algorithm may lead to either positive or negative time delay (i.e., late or early arrival of the symbol). In order to compensate for this delay post-FFT fine synchronization algorithms are used, which are based on the validity of Eq (1). For this reason, the OFDM symbol is extended by inserting both a cyclic postfix and prefix. The length of the cyclic postfix $L_{\text{post}}$ must be larger than the maximum expected negative delay of the symbol, while the length of the cyclic prefix $L_{\text{pre}}$ must be larger than the maximum expected delay plus the length of the equivalent CIR. This is necessary to ensure that any possible timing error will manifest itself as a linear phase rotation of the transmitted symbol and will not produce any Inter-Symbol Interference (ISI) terms. This special postfix/prefix-extended OFDM symbol structure is necessary only for the symbol(s) that will be used for synchronization purposes. The typical OFDM symbol structure with only cyclic prefix, is still used for the rest of the frame. Another possible way to apply the synchronization algorithms is to pre-delay the symbol by $L_{\text{pre}}$ and then use a cyclic prefix of length $L'_{\text{pre}} = L_{\text{pre}} + L_{\text{pro}}$.

From Eq. (1) it becomes obvious that, given a known channel and transmitted signal (Pilot-Symbol-Assisted-Modulation (PSAM) methods), one can estimate and compensate for the delay $\varepsilon$. Unfortunately, during the initial frame synchronization procedure, there is no information available about the channel. In order to circumvent this difficulty, three different philosophies can be followed, the applicability and accuracy of which depends on the channel characteristics.

I. Ideal Channel (IC) Method: This is a channel-blind method which presupposes that the channel is ideal.

II. Parametric Channel Modeling (PCM) Method: Under this method channels that have a non-negligible, easy-to-model, effect are assumed.

III. Unmodelable Channels (UC) Method: Under this method, effective modeling of the channel is considered too complex. In this case, non-linear techniques with increased complexity can be used in order to identify the channel characteristics and the timing error.

A. Two-Point-slope-estimation (TP) Algorithm

This is an algorithm that adopts the IC method, based on the fact that the time error produces a linear (in index $k$) phase rotation of the (post-FFT) observed symbols. We define

$$H_c(k) = \frac{Y_m(k)}{X_m(k)}; \quad 0 \leq k \leq N - 1$$

This is a deterministic transformation of the observed vector, since PSAM methods are assumed. Using Eq. (1), the above equation becomes:

$$H_c(k) = H_m(k)\exp(-j\alpha k) + n(k); \quad 0 \leq k \leq N - 1$$

From Eq. (3) it follows that $H_c(k)$ is the noisy estimate of the frequency response of the channel, when that is rotated by an amount proportional to the time error $\varepsilon$ and sub-carrier index $k$. If the noise is neglected and one further assumes that the effect of the channel on the phases of the observed vectors is independent of $k$, $\varepsilon$ can be estimated to be:

$$\hat{\varepsilon}_{1,\text{TP}} = C(\text{TP}) \sum_{k=k_1}^{k_2} \arg\frac{H_c(k+1)}{H_c(k)}$$

where $C(\text{TP}) = \frac{N}{2\pi(k_2 - k_1)}; \quad 0 \leq k_1 < k_2 \leq N - 2$

A close look at this algorithm reveals that the sums produced are telescopic ones. This means that the exploited information resides in the phases of the edge parameters $H_c(k+1)$ and $H_c(k_1)$ only. The rest of the symbols (and their phases) are only used for effective phase unwrapping. An equivalent realization of this algorithm would be to perform phase unwrapping with any other method. Then, the estimate of the time delay can be calculated as

$$\hat{\varepsilon}_{2,\text{TP}} = C(\text{TP})[\theta_u(k_2+1) - \theta_u(k_1)]$$

where $\theta_u(k)$ is the angle of $H_c(k)$ after phase unwrapping. These algorithms estimate the slope based on two points only (located at sub-carriers $k_1$ and $k_2+1$ respectively), thus they are less affected by noise as $k_2 - k_1$ becomes larger.

B. Mean-Square-Error-slope-estimation (MSE) algorithm
The **MSE algorithm** tries to make a better use of the set of observations by using the Mean-Square-Error estimation method. Using the assumption of a channel that produces a common phase rotation term only, $\theta_u(k)$ can be modeled as:

$$\theta_u(k) = c + ak + n(k) \quad (6)$$

where $c$ models precisely this the mean rotation and $n(k)$ is a zero-mean Gaussian distribution that models the possible small residual effect of the channel plus the effect of the thermal noise.

### E. First-Degree-Parametric-based-slope-estimation (FDP) algorithm

This algorithm adopts the PCM method. In order to counteract the common rotation term of the channel over the whole set of observables, these are transformed to:

$$\theta_u(k) = \theta_u(k + k_j) - \theta_u(k_i); \quad 0 \leq k \leq k_2 - k_1 \quad (7)$$

This modification takes out any common term but does not affect the linear rotation term, which carries the information about the timing error. Then $\theta_u^*(k)$ is modeled as:

$$\theta_u^*(k) = ak + bk^2 + n(k) \quad (8)$$

The second-order term approximates the effect of the channel on the observables. This approximation is based on the observation that 'mild' channels have a phase frequency-domain response that is smooth, which is mainly true for a small range of sub-carriers.

### F. Second-Degree-Parametric-based-slope-estimation (SDP) algorithm

Another way to manipulate the constant rotation is to consider it as a term to be jointly estimated. Thus, $\theta_u(k)$ can be modeled as:

$$\theta_u(k) = c + ak + bk^2 + n(k) \quad (9)$$

The MSE of the time error for any of the above algorithms, can be computed to be:

$$\hat{\varepsilon} = C_p S_0^0(a,b) + C_1 S_1^1(a,b) + C_2 S_2^2(a,b) \quad (10)$$

where we define

$$S_i^j(a,b) = \sum_{k=a}^b k^j \theta_{obs}(k) \quad (11)$$

with $\theta_{obs}(k)$ the corresponding observable vector. The $C_j$ coefficients for any algorithm are given in Table I.

The $K_i$ terms in this table are also defined to be:

$$K_i = K_i^c(a,b) = \sum_{k=a}^b k^i \quad (12)$$

### G. Third-Degree-Parametric-based-slope-estimation (TDP) algorithm

For most channels, models like the ones we have presented up to now cannot approximate the phase response of the channel for the whole spectrum (i.e., for the whole set of the sub-carriers). To efficiently do that, a partial approximation method is proposed. The observed vector of length $k_j - k_i + 1$ is split into I groups of sub-vectors, of equal length $N_{sv}$. Each, second-order models can be used to approximate partially each one of these sub-vectors. To implement an algorithm using this concept, we define

$$\theta_u^{(i)}(k) = \theta_u(iN_{sv} + k + k_1) - \theta_u(k + k_1) \quad (13)$$

with

$$0 \leq k \leq N_{sv} - 1; \quad 0 \leq i \leq I - 1$$

Then we model any $\theta_u^{(i)}(k)$ as

$$\theta_u^{(i)}(k) = ak + bhk^2 + n(k) \quad (14)$$

where $bhk^2$ is a square term that the channel probably induces for each of these sub-vectors Minimization of the metric

$$\sum_{i=0}^{I-1} \sum_{k=0}^{N_{sv}-1} (\theta_u^{(i)}(k) - ak - bhk^2)^2 \quad (15)$$

leads to

$$\hat{\varepsilon}_{TDP} = \frac{1}{I} \sum_{i=0}^{I-1} \left( C_1 \sum_{k=0}^{N_{sv}-1} bh^{(i)}(k) - C_2 \sum_{k=0}^{N_{sv}-1} h^{(i)}(k) \right) \quad (16)$$

where the corresponding constants are also described in Table I.

We note that all the channel-parametric algorithms are designed to counteract a common rotation of the post-FFT observables. Such a rotation may originate from the phase noise of the system or residual frequency offset ([10]).

### H. Energy-Detection (ED) Algorithm

The problem of identifying the phase rotation induced by timing error can be equivalently transformed to the problem of identifying the time of arrival of the first significant energy portion of the CIR. The noisy taps of the delayed CIR in the time domain can be identified by the Inverse-Fast–Fourier-Transform (IFFT):

$$h_m(l - \varepsilon) + n_g(l) = h_m'(l) + n_g(l) = \frac{1}{N} \sum_{k=0}^{N-1} H_k(k)e^{-j2\pi kld} \quad (17)$$

with

$$0 \leq l \leq N - 1,$$

where $n_g(l)$ is a noise term. This problem is equivalent to an energy detection problem in a noisy environment; thus, it amounts to the calculation of an optimal threshold $\Gamma$; when energy larger than the threshold arrives, the start of a frame is declared. Since the maximum is expected to be in the first arriving CIR samples, calculation of the whole IFFT is wasteful.
Thus, the sequential calculation of the IDFT outputs is proposed, followed by a decision on whether the energy of the corresponding tap is larger than the aforementioned threshold. If not, the algorithm continues with the next channel tap calculation, and so on. So, the algorithm can be described as follows:

\[
\hat{e}_{NL} = \varepsilon : \left\{ \frac{1}{N} \sum_{k=0}^{N-1} H_c(k) e^{\frac{2\pi i k}{N}} > \Gamma \right\}
\]  

for the first time. Notice that this algorithm can handle efficiently only positive time delay. Thus, according to the discussion in the beginning of this Section, it has to be utilized with schemes that pre-delay the received symbols.

I. Approximate-Energy-Detection (AED) Algorithm

In most cases, the length of the CIR of the equivalent channel plus the maximum expected delay is much smaller than the length of the OFDM symbol. This observation can be taken into consideration in order to decrease the number of the calculations that ED algorithm needs. As the time domain information is localized in the first time domain samples, it can be extracted also by the use of less then N frequency domain samples (i.e., by down-sampling the frequency domain vector). The down-sampling factor has to be such that the total frequency domain samples used for the CIR estimation, is larger than the length of maximum expected delayed CIR.

IV. SIMULATION RESULTS

In order to investigate the performance of the algorithms, simulations are performed for the 3 different types of channels. The Type-A channel is a mild one with negligible effect. The Type-B channel is of higher frequency selectivity with a non-negligible effect on the phase of the frequency domain samples. Finally, simulations are illustrated for a channel with high frequency selectivity (Type-C). These channels are used for simulations within the ADAMAS (IST-1999-10731) and WIND-FLEX (IST-1999-10025) projects. Only one pilot OFDM symbol is dedicated for fine-frame synch and it can be the same that is used for coarse. QPSK (4-QAM) modulation is employed. The length of the FFT is N=128. The noise is AWGN. The estimators use only half of the sub-carriers (N/2) for estimation. For this reason, only the AED algorithm has been included in the simulations. The ED algorithm is expected to have even better performance.

V. CONCLUSIONS

Several post-FFT Fine Frame Synchronization methods have been proposed and evaluated. The applicability of each depends on the channel characteristics. From the simulations it becomes apparent that the AED are the more robust methods. The channel-parametric algorithms that use simple algorithms (FDP, SDP) cannot provide reliable estimates (with bias less than half a sample) for channels of great frequency selectivity. TDP algorithms present performance comparable to the AED ones but seem to be more easily affected by the thermal noise. The proposed algorithms (when they are applicable) can produce reliable estimates even for one dedicated symbol only and even in the case of utilizing a portion of the sub-carriers.
### Table 1

<table>
<thead>
<tr>
<th>MSE</th>
<th>FDP</th>
<th>SDP</th>
<th>TDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a, b$</td>
<td>$k_1, k_2$</td>
<td>$0, k_1, k_2$</td>
<td>$k_1, k_2$</td>
</tr>
<tr>
<td>$C_0$</td>
<td>[ N \frac{K_1}{2\pi K_0 K_2 - K_1^2} ]</td>
<td>-</td>
<td>[ N \frac{K_1 K_2 - K_1}{2\pi K_0 K_2 K_4 + 2K_1 K_2 K_3 - K_1^2 K_4 - K_2^2 K_0 - K_3^2} ]</td>
</tr>
<tr>
<td>$C_1$</td>
<td>[ N \frac{K_0}{2\pi K_0 K_2 - K_1^2} ]</td>
<td>[ N \frac{K_1}{2\pi K_0 K_2 K_4 + 2K_1 K_2 K_3 - K_1^2 K_4 - K_2^2 K_0 - K_3^2} ]</td>
<td>[ N \frac{K_2}{2\pi K_2 K_4 - K_3^2} ]</td>
</tr>
<tr>
<td>$C_2$</td>
<td>-</td>
<td>[ N \frac{K_1}{2\pi K_2 K_4 - K_3^2} ]</td>
<td>[ N \frac{K_1 K_2 - K_0 K_3}{2\pi K_0 K_2 K_4 + 2K_1 K_2 K_3 - K_1^2 K_4 - K_2^2 K_0 - K_3^2} ]</td>
</tr>
</tbody>
</table>

**REFERENCES**


[2] ETSI TS 101 475 v1.2.1A (2000-04), "Broadband Radio Access Networks (BRAN); HIPERLAN Type 2; Physical (PHY) layer”.


