BER analysis of collaborative dual-hop wireless transmissions

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The error performance of collaborative dual-hop wireless transmissions with maximal-ratio combining diversity is presented. Specifically, using the well-known inequality between geometric and harmonic mean of positive random variables, an upper bound for the end-to-end signal-to-noise-ratio is derived, and it is used to efficiently evaluate the average error probability.

Introduction: Recently, relaying dual-hop transmissions have gained a new lease of life in collaborative/cooperative wireless communication systems [1, 2]. In collaborative diversity systems, intermediate mobile terminals are used to relay the signal between the base station and the destination mobile terminal, when the direct link is in deep fade. Scanning the up-to-date open technical literature, the number of published works concerning performance analysis of dual-hop wireless communications systems with collaborative diversity is relatively small. In [1], an outage probability formula is derived using the method of multi-user spatial diversity. Later, Hasna and Alouini studied the outage and the error performance of dual-hop systems with regenerative and non-regenerative relays over Nakagami-m [2] and Rayleigh-fading channels [3]. In this Letter, using the well-known inequality between geometric and harmonic mean of positive random variables (RVs), we derive an upper bound for the end-to-end signal-to-noise ratio (SNR), which is used to evaluate in closed-form an efficient and tight lower bound for the error performance of collaborative dual-hop transmissions using maximal-ratio-combining (MRC) diversity in the destination mobile terminal.

System model: A multi-user wireless communications system, where the source terminal S communicates with the destination terminal D through a direct link with SNR $\gamma_D$ and L dual-hop collaborative paths of non-regenerative (amplify and forward) relays, is considered in Fig. 1. Assuming MRC at the destination terminal, the overall SNR at the receiving end can be written as [2–4]:

$$\gamma_{end} = \gamma_o + \sum_{i=1}^{L} \frac{\gamma_S G_i}{\gamma_S + \gamma_D + 1}$$

(1)

where $\gamma_S$ is the instantaneous SNR between the source $S$ and relay $i$, and $\gamma_D$ is the instantaneous SNR between the destination $D$ and relay $i$.

Average error probability: The moment-generating function (MGF)-based approach [5, Chap. 1] for the performance evaluation of digital modulations over fading channels, allows us to obtain the average error probability for a wide variety of modulation schemes. Using (1), $\gamma_{end}$ can be rewritten as:

$$\gamma_{end} = \gamma_o + \sum_{i=1}^{L} \frac{1}{\gamma_S G_i} + \frac{1}{\gamma_D + 1} = \gamma_o + \sum_{i=1}^{L} H_i$$

(2)

where $H_i$ is the harmonic mean of the three positive RVs $\gamma_S$, $\gamma_D$, and $\gamma_o$, i.e. $H_i = (1/\gamma_S + 1/\gamma_D + 1/\gamma_o)^{-1}$ for any path.

Using the well-known inequality between harmonic and geometric mean of positive RVs [6, p. 45]

$$H_i \leq G_i$$

(3)

with $G_i$ being the geometric mean of $\gamma_S$, $\gamma_D$, and $\gamma_o$, i.e. $G_i = (\gamma_S \gamma_D \gamma_o)^{1/3}$, (2) results in:

$$\gamma_{end} \leq \gamma_o + \frac{1}{3} \sum_{i=1}^{L} \gamma_S \gamma_D \gamma_o^{2/3}$$

(4)

where $\gamma_o$ is now an upper bound of $\gamma_{end}$ having the advantage of mathematical tractability over that in (1). Owing to the independency of $\gamma_S$, $\gamma_D$, and $\gamma_o$, the MGF of $\gamma_o$ equals the product of MGFs as

$$M_{\gamma_o}(s) = \prod_{i=1}^{L} M_{\gamma_S}(s) M_{\gamma_D}(s)$$

(5)

where $M_{\gamma_S}(s)$ and $M_{\gamma_D}(s)$ are the MGFs of $\gamma_S$ and $1/3 \gamma_S \gamma_D \gamma_o^{2/3}$, respectively.

Owing to the MGF definition, $M_{\gamma_o}(s) \triangleq E(e^{s\gamma_o})$, (5) can be expressed as

$$M_{\gamma_o}(s) = \left( \frac{m_m}{\Gamma(m)} G_0 \right)^{m-1} e^{-m \gamma_o / G_0}$$

(7)

where $\Gamma(\cdot)$ is the gamma function [7, eqn. (8.310.1)], $\bar{\gamma}_o$ is the average SNR per hop and $m_i$ is the Nakagami parameter describing the fading severity of the ith hop and assumed, with no loss of generality, to be the same in all hops.

Using (6) and (7), the first integral in $I_i$, i.e. the one on $\gamma_S$, is of the form

$$I_1 = \frac{1}{\Gamma(m)} \left( \frac{m}{\bar{\gamma}_o} \right) \int_0^{\infty} \gamma_{end}^{m-1} G_i \left( \frac{m}{\bar{\gamma}_o} \right) d\gamma_{end}$$

(8)

where

$$G_m(a, b) \left( x \right) = \frac{x^a}{\beta_0 !} e^{bx}$$

is the Meijer’s G-function [7, Chap. 9.3] and $e^{-m \gamma_o / G_0} e^{m \gamma_o / G_0} e^{-m \gamma_o / G_0}$ are expressed in terms of the G-function [8]. Using [8], the integral $I_1$ can be evaluated in closed-form as:

$$I_1 = \frac{\sqrt{2} m^{1/2}}{(2\pi)^{1/2} \Gamma(m)} G_{2,3}^2 \left[ \frac{4 \bar{\gamma}_o^2}{3} \right]^{1/2} \int_0^{\infty} \left( \frac{1-m}{2} \right) G_{2,3} \left[ \frac{m}{\bar{\gamma}_o} \right] d\gamma_{end}$$

(9)

The second integral in $I_i$, i.e. the one on $\gamma_D$, can be solved in the same way as $I_1$, resulting in:

$$I_2 = \frac{\sqrt{2} m^{3/2}}{\pi^{1/2} \Gamma(m)} \left( \frac{m}{\bar{\gamma}_o} \right) \int_0^{\infty} \left( \frac{1-m}{2} \right) G_{2,3} \left[ \frac{m}{\bar{\gamma}_o} \right] d\gamma_{end}$$

(10)

Using the expression for the MGF of $\gamma_o$ [5], $M_{\gamma_o}(s)$ can be finally written as:
For identical links, i.e. $\gamma_i = \gamma_0 = \gamma_D = \gamma$ for $i = 1, 2, \ldots, L$, (11) can be written as:

$$M_i(s) = \left(1 - \frac{s^2 \gamma_0}{m}\right)^{-m} \prod_{i=1}^{L} \frac{\sqrt{32^{2m-1}}}{\pi^2 \Gamma^2(m)} G_{4,3}^3 \times \left(\frac{2^{s^2 \gamma_0}}{3^s m^s} \right)^{- \frac{1}{2}} \left(\frac{1}{2} \frac{2 - m}{2} \frac{1 - m}{2} \frac{2 - m}{2} \frac{1}{3} \frac{2}{3} \frac{2}{3} \right)^{\frac{1}{2}}$$

(12)

Having the MGF of $\gamma_k$ in closed-form, as given in (12), and using the MGF-based approach for the performance evaluation of digital modulations over fading channels [5, Chap. 1], the average bit and symbol error rate can be evaluated for a wide variety of M-ary modulations (such as M-ary phase-shift keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM)).

**Conclusions:** An efficient lower bound to the average BER performance of collaborative dual-hop wireless transmissions with MRC diversity in the destination terminal is presented, by applying the well-known inequality of geometric and harmonic mean of RVs. Numerical and simulation results show the tightness of the proposed bound.

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**References**


